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Identification of the number and locations of acoustic sources

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Abstract

The identification of the number and locations of acoustic sources is a fundamental problem of engineering acoustics. The method of multiple signal classification (MUSIC) has been widely used to solve this problem. The main drawback of the MUSIC method is that the unavoidable measurement noise should be identical for all the channels of measurement system. This requirement generally cannot be met in practical application. In this work, a method was proposed to identify the number and locations of multiple incoherent sources. The proposed method was based on the traditional MUSIC method, however, without the basic assumption about the identity of the measurement noise. Besides, a procedure was proposed to improve the calculation efficiency of identification. The numerical and experimental results demonstrated that the identification accuracy of the proposed method was far better than that of the traditional MUSIC method. The results also demonstrated that the proposed calculation procedure was very efficient, only the data at a few frequencies should be used for identification.

1. Introduction

The identification of the number and locations of acoustic sources is a fundamental problem in noise control. This subject has been investigated extensively in the past, and some methods have been proposed. Although the main function of near-field acoustic holography (NAH) is not to

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identify the number and locations of acoustic sources, one can find out the locations of sources from the identified particle velocity field [1–4]. However, NAH only functions well for sources with simple geometry; otherwise, NAH should be combined with the boundary element method to identify the sources with irregular geometry [5–7]. Besides, in NAH, the measurement must be very close to the sources, about one-half wavelength. This requirement often cannot be met in practice due to different reasons. For instance, the measurement sensor may affect the flow field if the sensor should be placed very close to the aeroacoustic source. Another problem is that if the locations of the sources are not known in advance, how can one place the sensor close to the sources within one-half wavelength?

If only the number, not the positions, of incoherent sources should be identified, the most powerful method is to find the rank of the cross-spectral matrix of the signals measured by pressure transducers [8,9]. The number of incoherent sources is equal to the rank of the cross-spectral matrix. This method is called cross-spectral matrix (CSM) analysis in this work for convenience. The locations of sources are more difficult to identify than the number of sources. As discussed by Schmidt [10], the method of multiple signal classification (MUSIC) is superior to the methods of ordinary beam forming, the maximum likelihood and the maximum entropy in the identification of the source positions. However, the main drawbacks of the MUSIC method are: (1) the sources should be incoherent, (2) the auto-spectrum of the measurement noise should be identical for all the measurement channels. In the past, the spatial-smoothing technique [11] has been used successfully to suppress the interference of multiple coherent signals. In other words, with the spatial-smoothing technique, the assumption of incoherence is no more a problem for the MUSIC method. However, the assumption about the auto-spectrum of the measurement noise is still a main limitation of the MUSIC method. One knows that this assumption generally cannot be fulfilled in the practical application. This limitation is due to the fact that the MUSIC method in principle is an eigenanalysis method. Only when the auto-spectrum of the measurement noise is identical for all the measurement channels, the eigenvalues of the cross-spectral matrix of measurement signals can be classified into two groups, i.e., the signal-plus-noise and noise segments. As best as we know, no report has been published to solve this limitation of the MUSIC method. Besides, because the MUSIC method is operated in the frequency domain, the source locations should be searched at every frequency of interest. It is very time consuming.

In this work, a method was proposed to identify the number and locations of multiple incoherent sources in space. The proposed method was based on the traditional MUSIC method, however, without the basic assumption about the identity of measurement noise. Besides, a procedure was proposed to improve the efficiency of finding the locations of sources. The accuracy and feasibility of the proposed method were verified by numerical simulation and experiment.

2. Theoretical formulation

Fig. 1 shows schematically the model of the problem investigated in this work. There are n incoherent sources distributed in the space, and there are m microphones to measure the radiated pressure. The number and locations of the sources are not known, and should be identified by

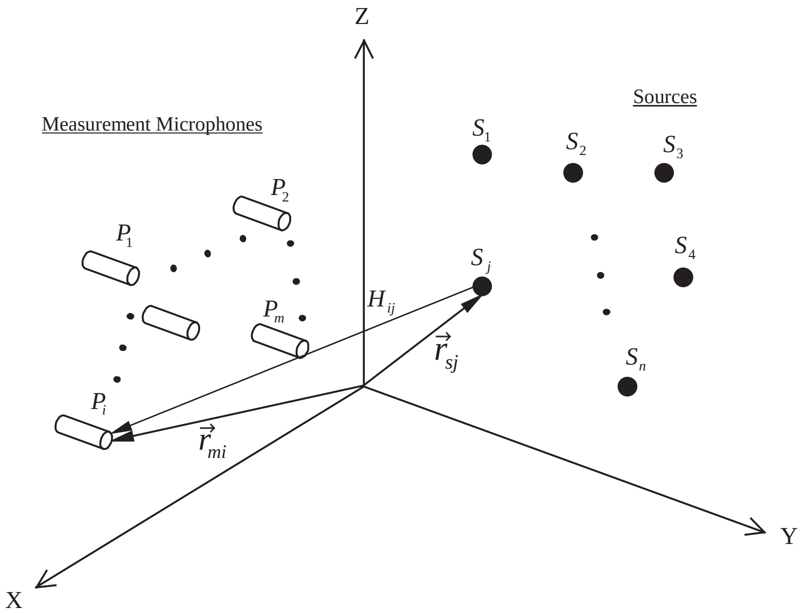

Fig. 1. Model of the investigated problem.

using the signals measured by the microphones. As mentioned, the method proposed in this work is based on the traditional MUSIC method. The traditional MUSIC method and the CSM analysis should be given briefly here in order to show clearly the connection with the proposed new method.

2.1. Identification of source number

As shown in Fig. 1, a vector $\{P\} = \{p_1, p_2, \ldots, p_\mathrm{m}\}^\mathrm{T}$ is used to represent the Fourier components (or spectra) of the pressure signals measured by the m microphones. As to how to determine the number of microphones, m, will be discussed later. Because the measurement noise is unavoidable, the $\{P\}$ vector can be written as

$$\{P\}_{m\times 1} = \{P_0\}_{m\times 1} + \{W\}_{m\times 1}, \tag{1}$$

where $\{P_0\}$ represents the exact value and $\{W\}$ represents the measurement noise. The cross-spectral matrix (or the covariance matrix) of the measured signals can be written as [8,9]

$$\left[C_p\right]_{m\times m} = \{P\}_{m\times 1} \cdot \{P\}^\mathrm{H}_{m\times 1} = \begin{bmatrix} c_{11,} & c_{12} & \cdots & c_{1m} \\ c_{21,} & c_{22} & \cdots & c_{2m} \\ \vdots & & & \vdots \\ c_{m1} & \cdots & \cdots & c_{mm} \end{bmatrix}_{m\times m}, \tag{2}$$

where the superscript H represents the complex conjugate of transpose. One can use singular value decomposition (SVD) to get the eigenvalues and eigenvectors of the $[C_p]$ matrix as

$$[C_p]_{m\times m} = [U_p][\Sigma_p][U_p]^{\mathrm{H}}, \tag{3}$$

where $[U_p]$ is an unitary matrix of eigenvectors and $[\Sigma_p]$ is a diagonal matrix containing the positive eigenvalues $\lambda_i, i = 1, \ldots, m$, in descending order. The rank of the $[C_p]$ matrix represents the number of principle components of that matrix, and thus represents the number of incoherent sources. If the number of measurement microphones, m, is larger than the number of incoherent sources, n, then the rank of the $[C_p]$ matrix should be less than m. Theoretically, the rank of a matrix is defined as the number of non-zero eigenvalues. However, the $\{P\}$ vector is perturbed by noise, so are the eigenvalues in Eq. (3). To find the exact number of incoherent sources, a criterion was proposed in this work to determine the true rank of the $[C_p]$ matrix from the result of SVD. Because the level of measurement noise depends on the instrument used for measurement and also on the noise level of measurement environment (or the background noise), the perturbation of the eigenvalues also depends on the above two factors. Consequently, the threshold value which is used to eliminate the "insignificant" eigenvalues caused by noise should be determined according to the measurement accuracy which one can realize in practice. In this work, the following procedure is proposed to find the true rank of the $[C_p]$ matrix:

(a) The eigenvalues of the $[C_p]$ matrix at all the frequencies of interest are found first by SVD. The maximum eigenvalue is indicated as $\lambda_{\max}$.
(b) The eigenvalues of the $[C_p]$ matrix at each frequency are normalized with respect to $\lambda_{\max}$, and only the normalized eigenvalues which are larger than 10^{-6} are accounted as the "true rank" of the $[C_p]$ matrix at that frequency. The true rank is function of frequency, and one uses N_i to represent the value of the true rank at the frequency f_i.

Because the $[U_p]$ matrix in Eq. (3) is an unitary matrix, the magnitude of the eigenvalues is directly related to the signal strength sensed by the microphones. The threshold value, 10^{-6}, defined in this work means approximately that the dynamic range of the measurement system is about 60 dB. Note that the main purpose of this work is to compare the difference between the traditional and the proposed modified MUSIC methods, the threshold value is not critical for this comparison because both methods use the same threshold value in this work. Note that the true rank N_i at the frequency f_i represents the number of sources which are active at the frequency f_i, it is different from the source number n defined in Fig. 1. For instance, there are three sources in space, but only a source radiates frequency f_i , then $n = 3$, $N_i = 1$. The source number n can be known only when all the positions of sources are identified, as discussed in Section 2.4.

If the rank of $[C_p]$ at all the frequencies of interest is less than the number of measurement microphones, the number of measurement microphones is sufficient to identify the number of sources. In other words, the minimum number of the measurement microphones is so determined to make to the rank of $[C_p]$ less than the number of microphones.

The above CSM method is not new, but the rank information from the CSM analysis will be used later with the modified MUSIC method to identify the source number n and also to improve the identification efficiency.

2.2. Identification of source locations

Before deriving the modified MUSIC method, the traditional MUSIC method should be given briefly in order to expose the main shortcoming of the traditional MUSIC method, and to show the connection with the modified method.

The so-called source strength (volume rate) is generally used to define the characteristic of simple point source [12]. The vector $\{P\}$ in Eq. (1) is related to the source strength vector $\{X\}$ by

$$\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ \vdots \\ p_m \end{Bmatrix}_{m\times 1} = \begin{bmatrix} H_{11,} & H_{12,} & \cdots & \cdots & H_{1n} \\ H_{21,} & H_{22,} & \cdots & \cdots & H_{2n,} \\ \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & & \vdots \\ H_{m1,} & H_{m2,} & \cdots & \cdots & H_{mn,} \end{bmatrix}_{m\times n} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{Bmatrix}_{n\times 1} + \begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_m \end{Bmatrix}_{m\times 1}, \tag{4}$$

where H_{ij} is the frequency response function (or transfer function) between the ith microphone and the jth source as shown in Fig. 1, x_i is the source strength of the ith source, w_i is the noise of the ith measurement channel.

Eq. (4) can be written in a compact form as

$$\{P\}_{m\times 1} = [H]_{m\times n}\{X\}_{n\times 1} + \{W\}_{m\times 1}. \tag{5}$$

Assume that the incident signals and the noise are uncorrelated, then the cross-spectral matrix can be written as

$$\left[C_p\right]_{m\times m} = [H]_{m\times n}\{X\}_{n\times 1}\{X\}^{\mathrm{H}}_{n\times 1}[H]^{\mathrm{H}}_{m\times n} + \{W\}_{m\times 1}\{W\}^{\mathrm{H}}_{m\times 1} \tag{6}$$

or in a compact form as

$$\left[C_p\right]_{m\times m} = [H]_{m\times n}[C_x]_{n\times n}[H]^{\mathrm{H}}_{m\times n} + [C_w]_{m\times m} \tag{7a}$$

$$= \left[U_p\right]\left[\Sigma_p\right]\left[U_p\right]^{\mathrm{H}}. \tag{7b}$$

Eq. (7b) is adopted directly from Eq. (3).

As discussed in Section 2.1, the number of active sources may vary with frequency. For simplicity, it was assumed that the number of active sources was n in deriving the traditional and the modified MUSIC methods. In other words, the rank of the noise-free $\left[C_p\right]$ matrix was considered as n in deriving the methods.

The traditional MUSIC method assumes that the noise at each measurement channel is mutually uncorrelated, and the auto-spectral density functions at all measurement channels are identical. With this assumption, the $[C_w]$ matrix is reduced to

$$[C_w] = \begin{bmatrix} \sigma & & & & 0 \\ & \sigma & & & \\ & & \sigma & & \\ & & & \ddots & \\ 0 & & & & \sigma \end{bmatrix}_{m\times m} = \sigma[I]_{m\times m}, \tag{8}$$

where σ is the value of the auto-spectral density function of measurement noise, and $[I]$ is a unit matrix. Note that the value σ is function of frequency.

With the assumption of Eq. (8), one can prove [10] that the last $(m-n)$ eigenvalues of Eq. (7b) are repeated, and are equal to σ, i.e.,

$$\lambda_{n+1} = \lambda_{n+2} = \cdots = \lambda_m = \sigma, \tag{9}$$

If the $(m-n)$ eigenvectors associated with the $(m-n)$ repeated eigenvalues are indicated as $\{E_k\}$, $k = n+1, n+2, \ldots, m$, then the eigenvectors $\{E_k\}$ can satisfy the following condition [10]:

$$[H]^{\mathrm{H}}_{m\times n}\{E_k\} = 0. \tag{10}$$

With the notation of Eq. (4), the $[H]$ matrix can be rewritten as

$$[H]^{\mathrm{H}} \equiv [\{h_1\}_{m\times 1}, \{h_2\}_{m\times 1}, \ldots, \{h_n\}_{m\times 1}]^{\mathrm{H}}_{m\times n}. \tag{11}$$

The $\{h_i\}$ vector represents the transfer function between the ith source and all the measurement microphones. The $\{h_i\}$ vector is function of frequency, the position of the ith source, and the positions of the measurement microphones. For the investigated problem, the positions of microphones are known, the only unknowns are the source positions. Therefore, the result of Eq. (10) can be used to find the source positions. To find the source positions, one can arbitrarily select a trial field point (indicated as "j" point), and then calculate the transfer function $\{\bar{h}_j\}$ between the trial field point and all the measurement microphones. The traditional MUSIC method uses the $\{\bar{h}_j\}$ vector to find a power function P_m as

$$P_m(\mathrm{j}, \omega) \equiv \frac{1}{\sum_{k=n+1}^{m} \left(\{\bar{h}_j\}^{\mathrm{H}}\{E_k\}\right)^2}, \quad j = 1, 2, \ldots \ell, \tag{12}$$

where ℓ indicates the number of trial field points, and ω is the frequency.

One knows that if the trial field point is the location of source, then theoretically the $\{\bar{h}_j\}$ should be equal to one of the $\{h_i\}$ in Eq. (11), and the value of $P_m(\mathrm{j}, \omega)$ should be infinitely large (according to the result of Eq. (10)). In other words, the positions of peaks of the $P_m(\mathrm{j}, \omega)$ distribution represent the positions of sources.

The above derivation is the traditional MUSIC method. From the above derivation one can find two main problems,

(1) The result of Eq. (10) is based on the assumption of Eq. (8). This assumption generally cannot be fulfilled in practice because the noise includes the part which is sensed along with the signals and the part which is generated internally by the measurement instrument.

(2) The calculation of the $P_m(\mathrm{j},\omega)$ is very time consuming. For instance, if the investigated space is divided into one thousand trial field points, and there are two thousands discrete frequencies in the frequency range of interest, then one should first calculate one thousand $\{\bar{h}_j\}$ vectors, and calculate the $P_m(\mathrm{j},\omega)$ function 2×10^6 times (i.e., 2000×1000).

In this work, the MUSIC method was modified to solve the first problem and a new calculation procedure was proposed to enhance the calculation efficiency.

2.3. Modified MUSIC method

Eq. (7a) can be rewritten as

$$
\begin{aligned}
\left[C_p\right]_{m\times m} &= [H][C_x][H]^{\mathrm{H}} + \begin{bmatrix} \sigma_1 & & & & 0 \\ & \sigma_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & \sigma_m \end{bmatrix}_{m\times m} \\
&\equiv [C_{xH}]_{m\times m} + \begin{bmatrix} \sigma_1 & & & & 0 \\ & \sigma_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & \sigma_m \end{bmatrix}_{m\times m},
\end{aligned}
\tag{13}
$$

where $\sigma_1 \neq \sigma_2 \neq \cdots \neq \sigma_m$, represent the auto-spectra of the measurement noise.

The first term in the right-hand side of Eq. (13) can be decomposed into eigenvalues and eigenvectors by the SVD, and Eq. (13) becomes

$$
\left[C_p\right]_{m\times m} = [U_x]\begin{bmatrix} \alpha_{1,} & & & & & 0 \\ & \alpha_{2,} & & & & \\ & & \ddots & & & \\ & & & \alpha_n & & \\ & & & & 0 & \\ & & & & & \ddots & \\ 0 & & & & & & 0 \end{bmatrix}[U_x]^{\mathrm{H}} + \begin{bmatrix} \sigma_1 & & & & 0 \\ & \sigma_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & \sigma_m \end{bmatrix}, \tag{14}
$$

where $[U_x]$ represents the unitary matrix of eigenvectors of $[C_{xH}]$ matrix and $\alpha_1, \alpha_2 \cdots \alpha_n$ represent the positive eigenvalues in descending order. Note again that it was assumed that the number of active sources was n in deriving the method.

The basic concept of the modified method is to derive a method which can adjust the measurement noise levels into equal level according to the eigenvalues of the $[C_p]$ matrix. In other words, one can add an artificial noise to the measured $[C_p]$ matrix to make the noise level at each measurement channel to be equal. The proposed method is based on an important property of eigenanalysis of the $[C_p]$ matrix found in this work. The property is called "Property A" in the following discussion. As mentioned, the number of active sources (or the true rank) may vary with frequency. For simplicity, the number of active sources was assumed as n in discussing the "Property A".

Property A. Assume that σ_k is the largest value among the $\sigma_1, \sigma_2, \ldots, \sigma_m$ in Eq. (14), and if an arbitrary positive value is added to σ_k, then $\bar{\lambda}_{n+1}/\bar{\lambda}_m \geqslant \lambda_{n+1}/\lambda_m$.

Proof. Note that the λ_i represent the eigenvalues of the $[C_p]$ matrix in descending order, i.e., $\lambda_1 > \lambda_2 > \cdots > \lambda_m$, as indicated in Eq. (3). Adding an arbitrary positive value to σ_k in Eq. (14) is the same as adding the positive value to the kth diagonal term of the $[C_p]$ matrix. The eigenvalues of the original $[C_p]$ matrix are indicated as λ_i, and the eigenvalues of the $[C_p]$ matrix with a positive value added to the kth diagonal term of the $[C_p]$ matrix are indicated as $\bar{\lambda}_i$. Mathematically there are two possible results after adding a positive value to σ_k, i.e.,

$$
\begin{aligned}
&\bar{\lambda}_{n+1}/\bar{\lambda}_m < \lambda_{n+1}/\lambda_m \qquad \text{result}(a),\\
&\bar{\lambda}_{n+1}/\bar{\lambda}_m \geqslant \lambda_{n+1}/\lambda_m \qquad \text{result}(b).
\end{aligned}
\tag{15}
$$

As proved in Appendix A, result (a) is impossible. Result (b) is the correct result. □

One can use the result of Property A to adjust the diagonal terms of the $[C_p]$ matrix to make the noise levels to be equal and cause the last $(m-n)$ eigenvalues to be repeated. The λ_{n+1} and λ_m represent the largest and smallest eigenvalues induced by measurement noise, and are called the largest and smallest noise-eigenvalues in this work. If the number of active sources (or the true rank) is not n, then the value n in Eq. (15) should be replaced by the number of active sources. The basic procedure to adjust the diagonal terms of the $[C_p]$ matrix is to add (may be many times) a small positive value to the first diagonal term of the $[C_p]$ matrix, and check the ratio of the largest noise-eigenvalue to the smallest noise-eigenvalue at each time of adding. When the ratio becomes increasing (i.e., the characteristic of result (b) in Eq. (15)), the first diagonal term becomes the term having the largest measurement noise, and a small positive value should add to the second diagonal term, etc. The adjustment can converge quickly by a simple do-loop program. The detailed procedure to adjust the diagonal terms of the $[C_p]$ matrix is shown in Fig. 2. One can find that the smallest eigenvalue, λ_m, of the original $[C_p]$ matrix is used as the initial positive value, δ, for adjustment. This is because the result of Eq. (9) gives us a hint that the last $(m-n)$ eigenvalues are approximately the same order as the noise level. Thus, the λ_m is a proper initial value to adjust the measurement noise levels. Of course, theoretically one can use any arbitrary positive value for the adjustment. However, if the positive value is too small, then the convergent time will be too long. On the contrary, if the positive value is too large, one cannot obtain a fine tune. As shown in the left side of Fig. 2, the positive value, δ, used for adjustment will automatically become smaller and smaller after each turn of adjustment (i.e., from the first to the last diagonal terms) in order to obtain a very fine tune. The convergent criterion used in this work is that the ratio of the largest noise-eigenvalue to the smallest noise-eigenvalue should be smaller than 1.0001. When the adjustment is converged, the conditions of Eqs. (9) and (10) can be satisfied, and Eq. (12) can be

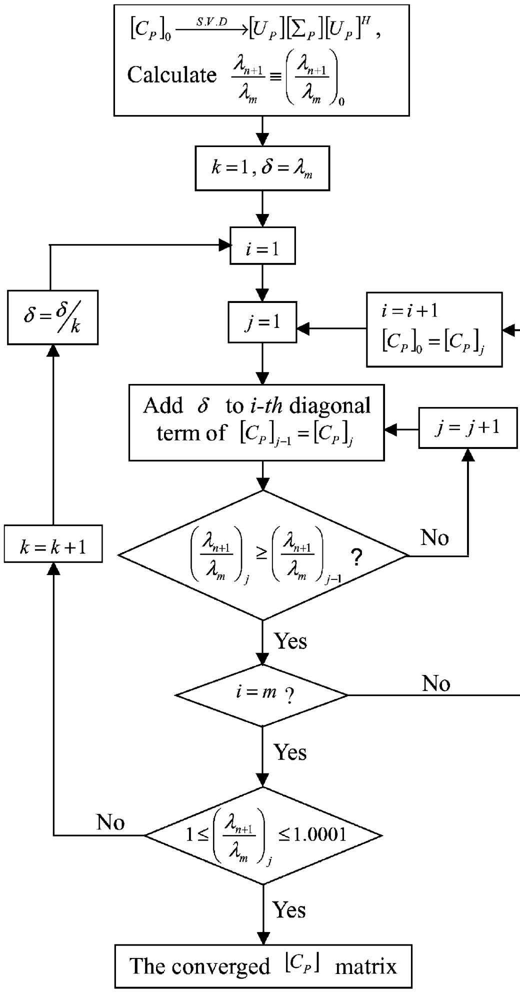

Fig. 2. Flow diagram to adjust the $[C_p]$ matrix.

used to identify the source positions. In other words, the modified method proposed in this work uses the converged $[C_p]$ matrix to calculate the P_m value in Eq. (12).

Note that the $[C_p]$ matrix is function of frequency. If the $[C_p]$ matrix should be adjusted at each frequency of measurement, it will be very time consuming. In this work, a procedure was proposed to select only a few "important" frequencies for identification so that the adjustment of the $[C_p]$ matrix would not be a burden for the modified MUSIC method. This procedure will be explained in the next section.

2.4. Calculation procedure with high efficiency

Both the traditional and the modified MUSIC methods use the P_m value in Eq. (12) to identify the source positions. As mentioned, the calculation of the P_m value is very time consuming because the P_m value is function of trial field points and frequencies. A new calculation procedure with high efficiency was proposed in this work. Instead of calculating the P_m value at all frequencies, the basic concept of the proposed procedure is to select only a few "important" frequencies to identify the number and positions of sources. This is especially important for the modified method because the modified method needs to adjust the $[C_p]$ matrix only at these important frequencies. How to select the "important" frequencies will be explained in the following calculation procedure. The proposed calculation procedure is explained step by step in what follows.

(1) Calculate the individual power spectrum of the signal measured by the individual microphone.
(2) Calculate an average power spectrum from the m spectra of step (1).
(3) Calculate the envelope of the average power spectrum first, and then find the frequencies of local maxima of the envelope. The frequencies of local maxima are indicated as $f_1, f_2, \ldots, f_k$.
(4) Plot the distribution of the true rank with frequency. The distribution has some local maxima, and the frequencies of local maxima are indicated as $f_{k+1}, f_{k+2}, \ldots, f_p$.
(5) The values of the true rank at frequencies f_1, $f_2, \ldots, f_{\mathrm{p}}$ can be found from the true rank distribution in step (4), and are indicated as N_1, $N_2, \ldots, N_p$.
(6) Calculate the spatial distribution of $P_m(\mathrm{j}, \omega_1)$ at frequency f_1. From the distribution of $P_m(\mathrm{j}, \omega_1)$ some local maxima can be found. The largest N_1 maxima are regarded as the locations of sources. The locations are indicated as $S_{1,1}$, $S_{1,2}, \ldots, S_{1,N1}$.
(7) Repeat the calculation of step (6) for other frequencies, $f_2, \ldots, f_p$.
(8) All the locations obtained from frequencies $f_1, \ldots, f_p$ represent the locations of sources. Because the locations of some sources may be identified repeatedly from different frequencies, if there are only n different values of $S_{i,j}$ from all the $S_{i,j}$, then the total number of sources must be n.

The above eight steps are the proposed calculation procedure. The traditional MUSIC method uses the original $[C_p]$ matrix to calculate P_m distribution while the modified MUSIC method used the converged $[C_p]$ matrix to calculate the P_m distribution. One can understand the calculation procedure more clearly only via examples in the next section; however, the reasons for some steps should be explained here first. Because the measurement microphones generally are arranged at the locations at which the source signals can be clearly measured by the microphones, the average power spectrum obtained at step (2) should be highly correlated with the spectra of the sources. The frequencies $f_1, f_2, \ldots, f_k$ obtained at step (3) can be regarded as the main frequencies of the sources. The reason for step (4) is to find the other main frequencies of sources which may be not found at step (3). For instance, the spectrum of a weak source may be overlapped (or covered) by the spectrum of a strong source so that the characteristic of the weak source may not show clearly in the envelope of the average spectrum. This is somewhat complicate, and will be explained further with an example in the next section. Because the values of $P_m(\mathrm{j}, \omega)$ cannot be used directly to judge the location of sources, the purpose of step (6) is to use the discrete data of $P_m(\mathrm{j}, \omega)$ to generate the spatial distribution of $P_m(\mathrm{j}, \omega)$ in order to find the largest N_i local maxima. Note that the N_i value should be determined first in step (5).

3. Simulation results and discussion

Before discussing the experimental results, some examples of numerical simulation will be used to show the effect of measurement noise on the identification results and to demonstrate the advantage of the proposed method.

3.1. Form of measurement noise

Theoretically, one can assume an arbitrary noise to simulate the measurement noise. However, in order to simulate the practical condition more realistically, a noise distribution which was obtained from a practical measurement system was used in the numerical simulation. A measurement system which included the microphone, amplifier and a spectrum analyzer was set-up in an anechoic chamber. Without any source signal, an averaged power spectrum was measured by the measurement system. The spectrum thus includes the noise of measurement environment (or background noise) and the noise generated internally by the measurement system. A typical power spectrum of noise is shown in Fig. 3. One can find that the spectrum is fairly flat above 200 Hz. The relatively large value of noise below 200 Hz is mainly due to the environment noise. The noise spectrum of Fig. 3 was multiplied by five arbitrary constants to simulate the five noise spectra measured by the five channels of measurement system. A detailed signal-to-noise ratio will be given in each example.

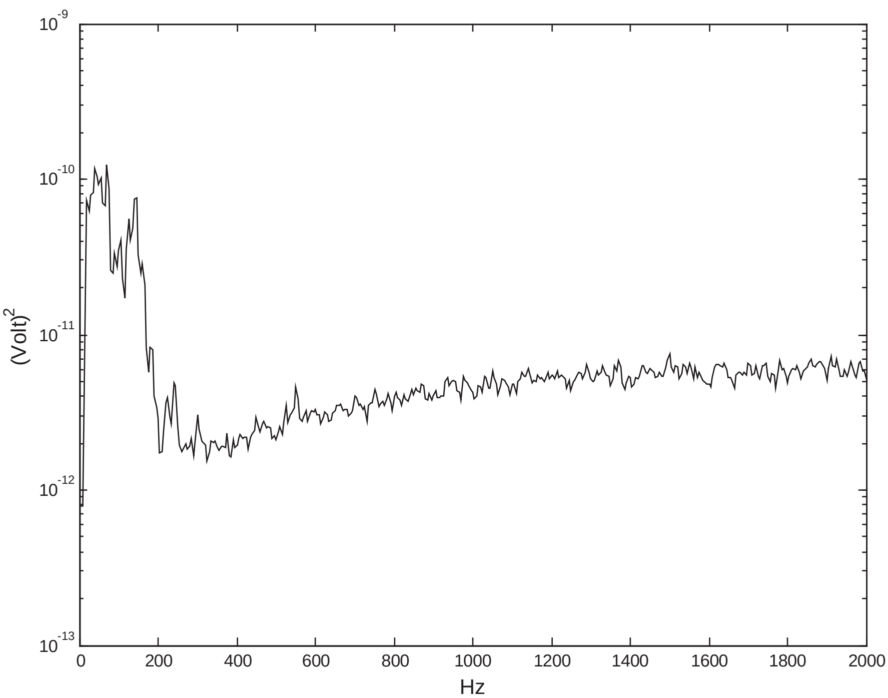

Fig. 3. A typical power spectrum of measurement noise.

3.2. Example 1, one-dimensional case

In order to show clearly the difference between the traditional MUSIC method and the modified method, a one-dimensional case was discussed first because the distribution of $P_m(\mathrm{j},\omega)$ function could be plotted clearly in one-dimensional case. The model of the one-dimensional case is shown in Fig. 4. There are three-point sources located in the X-axis and five measurement microphones are located in the X–Y plane. The detailed coordinates of sources and microphones are given in Table 1. As mentioned, source strength (volume rate) is generally used to define the characteristic of simple point source [12]. The spectra of the source strength are shown in Fig. 5. Basically the sources are narrow band random signals with central frequencies at 400, 800 and 1000 Hz, respectively. According to the proposed calculation procedure in Section 2.4, the envelope of the average spectrum is shown in Fig. 6. The frequencies of the local maxima are $f_1 = 405\,\text{Hz}$, and $f_2 = 770\,\text{Hz}$, as indicated in Fig. 6. The true rank distribution of the $[C_p]$ matrix is shown in Fig. 7, and the frequencies of the local maxima are $f_3 = 520\,\text{Hz}$, $f_4 = 1120\,\text{Hz}$. Note that the rank distribution is calculated from the noise contaminated signals, and then truncated according to the criterion proposed in this work. The signal-to-noise ratio will be given later. According to the proposed calculation procedure, only the data at these four frequencies, i.e., 405, 520, 770, 1120 Hz should be used to identify the number and locations of the sources.

With this example, the reason for the step (4) of the calculation procedure can be explained more clearly. As shown in Fig. 5, the main frequencies of source 3 are about from 800 to 1200 Hz. However, the envelope of average spectrum cannot reflect the main frequencies of source 3 because the spectrum of source 3 is relatively small and is overlapped (or covered) by source 2. However, the frequency f_4 in Fig. 7 can make up the overlook in the envelope of the average spectrum.

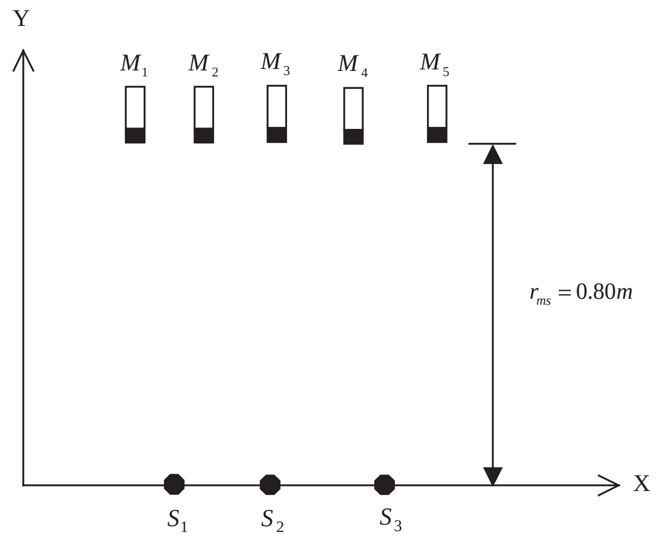

Fig. 4. The simulated example, one-dimensional case.

Table 1
Positions of sources and microphones of example 1

	Position vectors	(x, y) (m)
Point sources	$\vec{r}_{s1}$	(0.221, 0)
	$\vec{r}_{s2}$	(0.331, 0)
	$\vec{r}_{s3}$	(0.451, 0)
Measurement microphones	$\vec{r}_{m1}$	(0.20, 0.80)
	$\vec{r}_{m2}$	(0.30, 0.80)
	$\vec{r}_{m3}$	(0.40, 0.80)
	$\vec{r}_{m4}$	(0.50, 0.80)
	$\vec{r}_{m5}$	(0.60, 0.80)

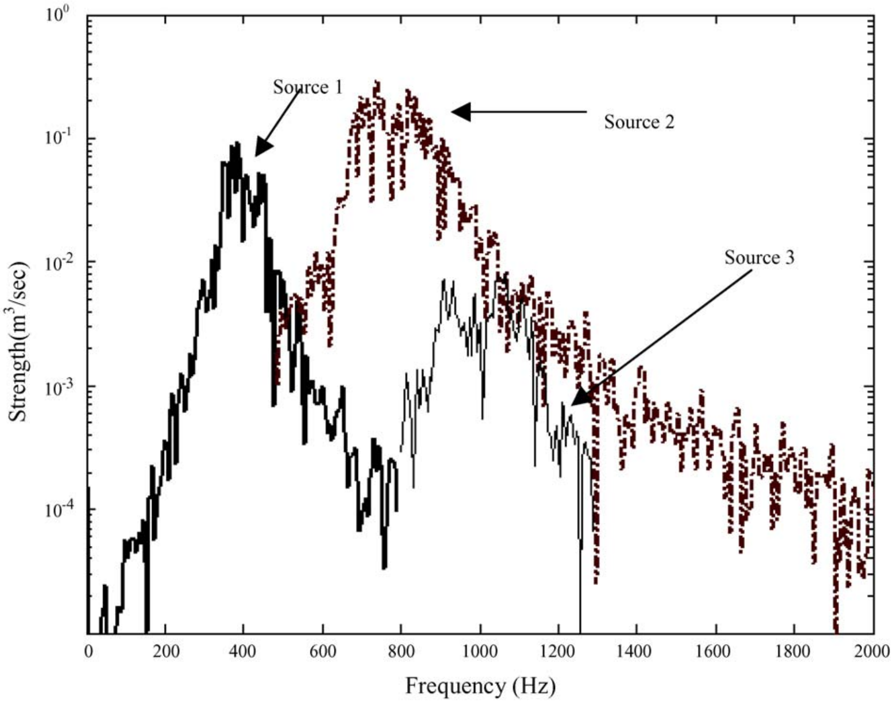

Fig. 5. Spectra of the three simulated sources.

Although the simulated measurement noise is distributed from 0 to 2000 Hz, only the noise levels at the selected four frequencies may affect the identification result. Table 2 shows the signal-to-noise ratios (SNR) at these four frequencies. The SNR is

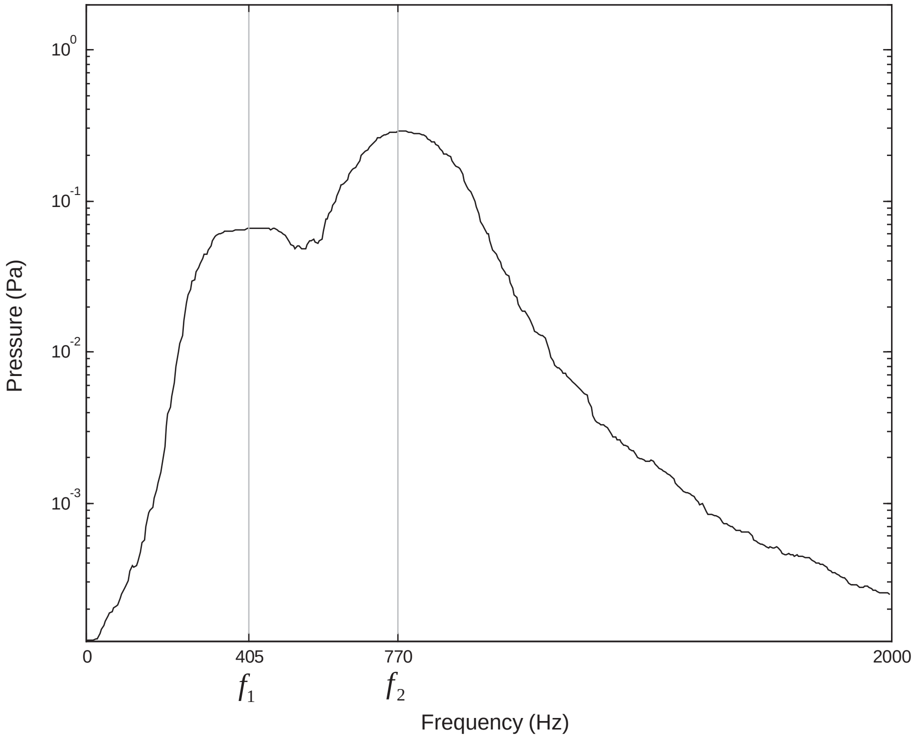

Fig. 6. Envelope of average spectrum, example 1, f_1, f_2 are frequencies of local maxima.

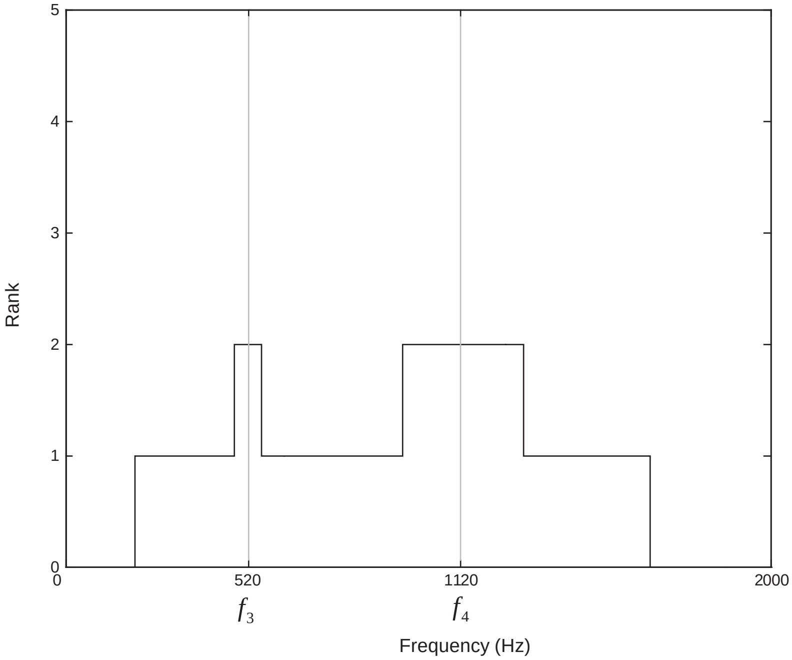

Fig. 7. The true rank distribution with local maxima at f_3, f_4.

Table 2
The signal-to-noise ratios at the selected frequencies

	405 Hz		520 Hz		770 Hz		1120 Hz	
	Noise	SNR (dB)	Noise	SNR (dB)	Noise	SNR (dB)	Noise	SNR (dB)
Mic. 1	$\sigma_1 = 2.1e-9$	62.0	$\sigma_1 = 3.4e-10$	38.5	$\sigma_1 = 3.7e-9$	76.4	$\sigma_1 = 3.0e-11$	48.6
Mic. 2	$\sigma_2 = 1.6e-8$	53.1	$\sigma_2 = 3.5e-9$	28.4	$\sigma_2 = 1.4e-7$	60.7	$\sigma_2 = 4.6e-9$	27.6
Mic. 3	$\sigma_3 = 1.8e-9$	62.4	$\sigma_3 = 2.6e-9$	29.5	$\sigma_3 = 1.7e-7$	59.9	$\sigma_3 = 1.9e-9$	32.1
Mic. 4	$\sigma_4 = 4.3e-10$	68.3	$\sigma_4 = 1.2e-11$	52.7	$\sigma_4 = 1.9e-5$	69.2	$\sigma_4 = 1.8e-10$	42.7
Mic. 5	$\sigma_5 = 1.0e-9$	64.0	$\sigma_5 = 1.8e-10$	40.6	$\sigma_5 = 3.1e-10$	86.8	$\sigma_5 = 1.0e-10$	45.0

defined as

$$\mathrm{SNR}(\omega_i) = 10\log\left(\frac{P_s(\omega_i)}{P_N(\omega_i)}\right), \tag{16}$$

where P_s and P_N represent the magnitude of the auto-spectra of signal and noise at frequency ω_1, respectively. The absolute magnitude of noise is also shown in Table 2. One can find that the noise values are different in each measurement microphone. The P_m distributions obtained from the traditional MUSIC method at frequencies f_1 to f_4 are shown in Fig. 8(a)–(d). The spatial resolution (or the mesh) used for calculating the P_m distributions is 1 mm along the source line, $Y = 0$. Because the P_m distributions were calculated at the source line, theoretically the P_m distributions should have sharp peaks at the positions of sources. One can find that the positions of peaks deviate from the positions of sources or there is no local peak in the P_m distribution. The source positions identified by the traditional MUSIC method are listed in Table 3. The results of Fig. 8 and Table 3 show clearly the main problem of the traditional MUSIC method. The P_m distributions obtained from the modified MUSIC method are shown in Fig. 9(a)–(d). Comparing the results of Figs. (8) and (9) one can find clearly the difference between the traditional and the modified methods. Note that the peak values at Fig. 9 are very large while the peak values at Fig. 8 are relatively small.

The positions of sources identified by the modified MUCIS method are listed in Table 4. One can find that the source positions can be identified exactly by the modified MUSIC method. From the result of Table 4 one also finds that the number of sources is three because only three different positions are found from the four frequencies.

3.3. Example 2, two-dimensional case

In this example, the three sources are located in a plane, and the detailed coordinates of the sources and microphones are listed in Table 5. The signals are the same as example 1. According to the same procedure like example 1, one finds four frequencies, i.e., 410, 525, 790, 1115 Hz, to identify the number and positions of sources. The absolute noise values and the SNR at these four frequencies are given in Table 6. To find the positions of sources, the P_m values at all trial field points should be calculated first and the P_m distribution is then used to find the local maxima. One cannot find the source positions directly from the P_m values or from the distribution on a single

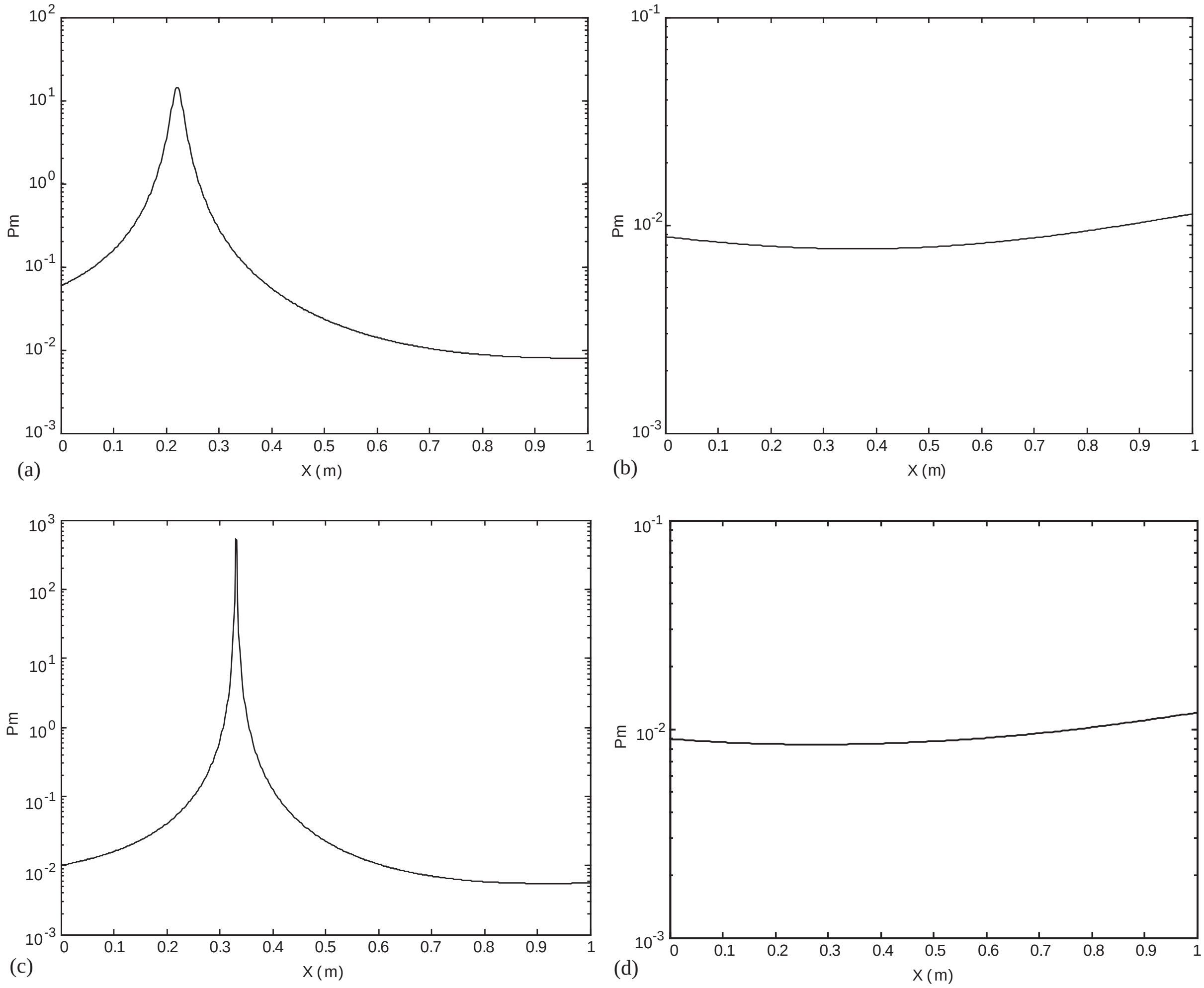

Fig. 8. P_m distributions calculated by the traditional MUSIC method, (a) $f_1 = 405\,\text{Hz}$, (b) $f_2 = 520\,\text{Hz}$, (c) $f_3 = 770\,\text{Hz}$, (d) $f_4 = 1120\,\text{Hz}$.

Table 3
Source positions identified by the traditional MUSIC method

Frequency (Hz)	Identified source position (x, y) (m)		
405	$S_{11}(0.232, 0)$		
520	No local maximum		
770	$S_{31}(0.388, 0)$		
1120	No local maximum		
Summary of result			
Exact (x, y)	(0.221, 0)	(0.331, 0)	(0.451, 0)
Identified (x, y)	(0.232, 0)	(0.338, 0)	—

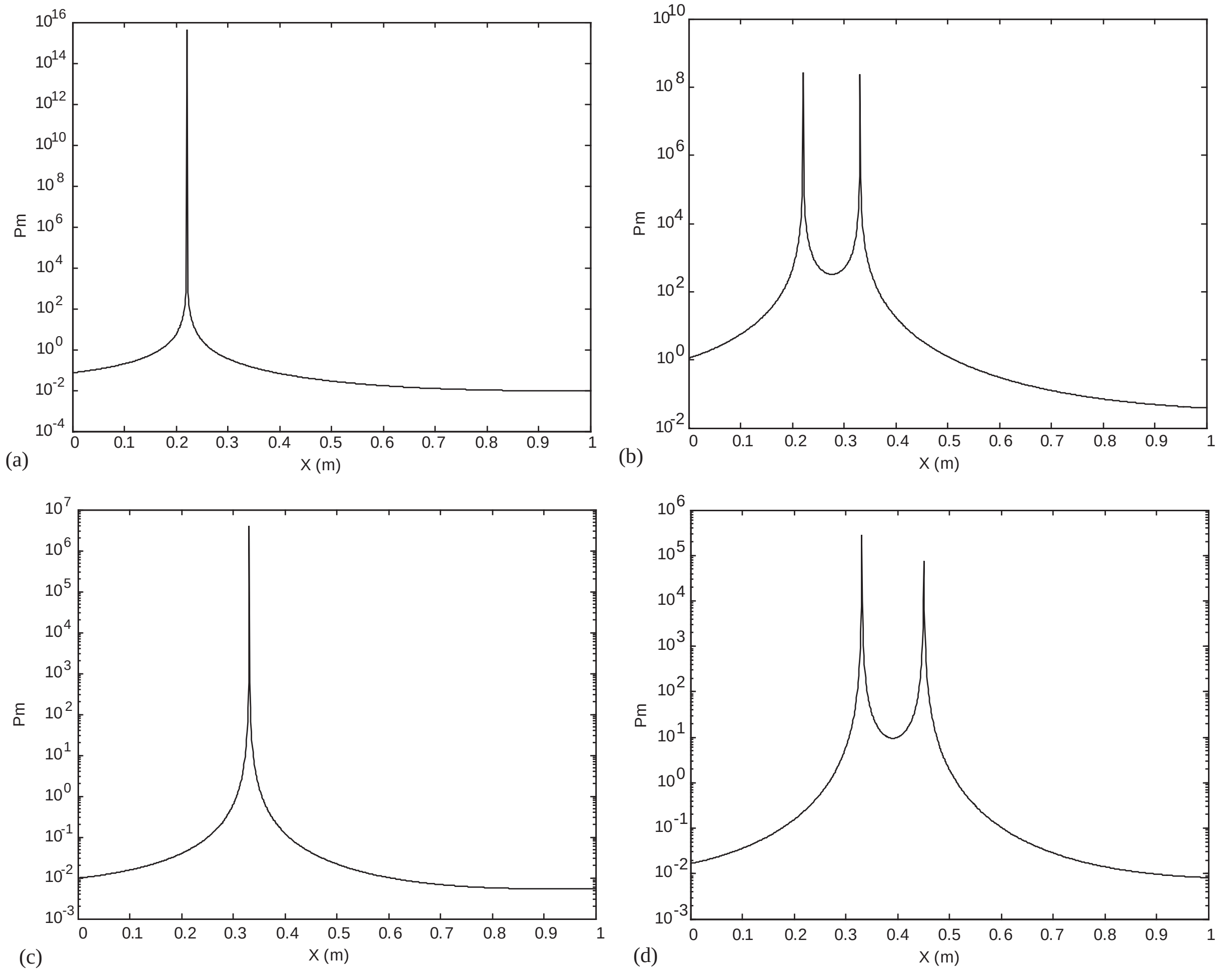

Fig. 9. P_m distributions calculated by the modified method, (a) $f_1 = 405\,\text{Hz}$, (b) $f_2 = 520\,\text{Hz}$, (c) $f_3 = 770\,\text{Hz}$, (d) $f_4 = 1120\,\text{Hz}$.

Table 4
Source positions identified by the modified method

Frequency (Hz)	Identified source position (x, y) (m)		
405	S_{11}(0.221, 0)		
520	S_{21}(0.221, 0), S_{22}(0.331, 0)		
770	S_{31}(0.331, 0)		
1120	S_{41}(0.331, 0), S_{42}(0.451, 0)		
Summary of result			
Exact (x, y)	(0.221, 0)	(0.331, 0)	(0.451, 0)
Identified (x, y)	(0.221, 0)	(0.331, 0)	(0.451, 0)

Table 5
Positions of sources and microphones of example 2

	Position vectors	(x, y, z) (m)
Point sources	$\vec{r}_{s1}$	(0.44, 0.70, 0.22)
	$\vec{r}_{s2}$	(0.33, 0.70, 0.33)
	$\vec{r}_{s3}$	(0.22, 0.70, 0.44)
Measurement microphones	$\vec{r}_{m1}$	(0.20, 0.00, 0.20)
	$\vec{r}_{m2}$	(0.40, 0.00, 0.20)
	$\vec{r}_{m3}$	(0.30, 0.10, 0.30)
	$\vec{r}_{m4}$	(0.20, 0.00, 0.40)
	$\vec{r}_{m5}$	(0.40, 0.00, 0.40)

Table 6
The signal-to-noise ratios at the selected frequencies, example 2

	410 Hz		525 Hz		790 Hz		1115 Hz	
	Noise	SNR (dB)	Noise	SNR (dB)	Noise	SNR (dB)	Noise	SNR (dB)
Mic. 1	$\sigma_1 = 7.7e-10$	57.6	$\sigma_1 = 1.3e-8$	21.4	$\sigma_1 = 3.5e-9$	58.3	$\sigma_1 = 4.7e-10$	38.1
Mic. 2	$\sigma_2 = 5.1e-11$	69.7	$\sigma_2 = 8.4e-10$	34.1	$\sigma_2 = 2.0e-9$	60.9	$\sigma_2 = 3.6e-10$	37.4
Mic. 3	$\sigma_3 = 6.7e-9$	49.7	$\sigma_3 = 5.3e-9$	27.0	$\sigma_3 = 1.2e-8$	54.6	$\sigma_3 = 1.9e-8$	23.4
Mic. 4	$\sigma_4 = 4.2e-10$	59.9	$\sigma_4 = 2.6e-10$	38.3	$\sigma_4 = 3.4e-10$	68.5	$\sigma_4 = 5.5e-10$	38.7
Mic. 5	$\sigma_5 = 4.5e-10$	60.0	$\sigma_5 = 2.4e-9$	29.1	$\sigma_5 = 4.6e-10$	67.4	$\sigma_5 = 1.2e-9$	34.0

plane. However, theoretically the P_m distribution on the source plane (i.e., a source is located on that plane) should have a sharp local maximum at the position of source. In this example, the three sources are located on the same plane ($Y = 0.7\,\text{m}$), theoretically the source positions can be easily identified from the P_m distribution on this source plane. Fig. 10(a)–(d) shows the P_m distribution on the source plane obtained from the traditional MUSIC method. The spatial resolution (or the mesh of the trial field points) used for the P_m calculation is 1 cm. Only two local maxima were found from these distributions. The source positions identified by the traditional MUSIC method are listed in Table 7. Only two source positions can be identified by the traditional method. The P_m distributions on the source plane obtained from the modified MUSIC method are shown in Fig. 11(a)–(d). The source positions can be clearly identified from the P_m distributions. The results identified by the modified method are given in Table 8. Because all the source positions are exactly located at the mesh points used for calculation, the source positions can be exactly identified by the modified method. Of course, if the source positions are not exactly located at the mesh points, an error with one mesh resolution may be found. The result of Table 8 also indicates that the number of sources is three. It should be pointed out that the peak values of Fig. 10 are relatively small (or not clear) while the peak values of Fig. 11 are very large (very sharp).

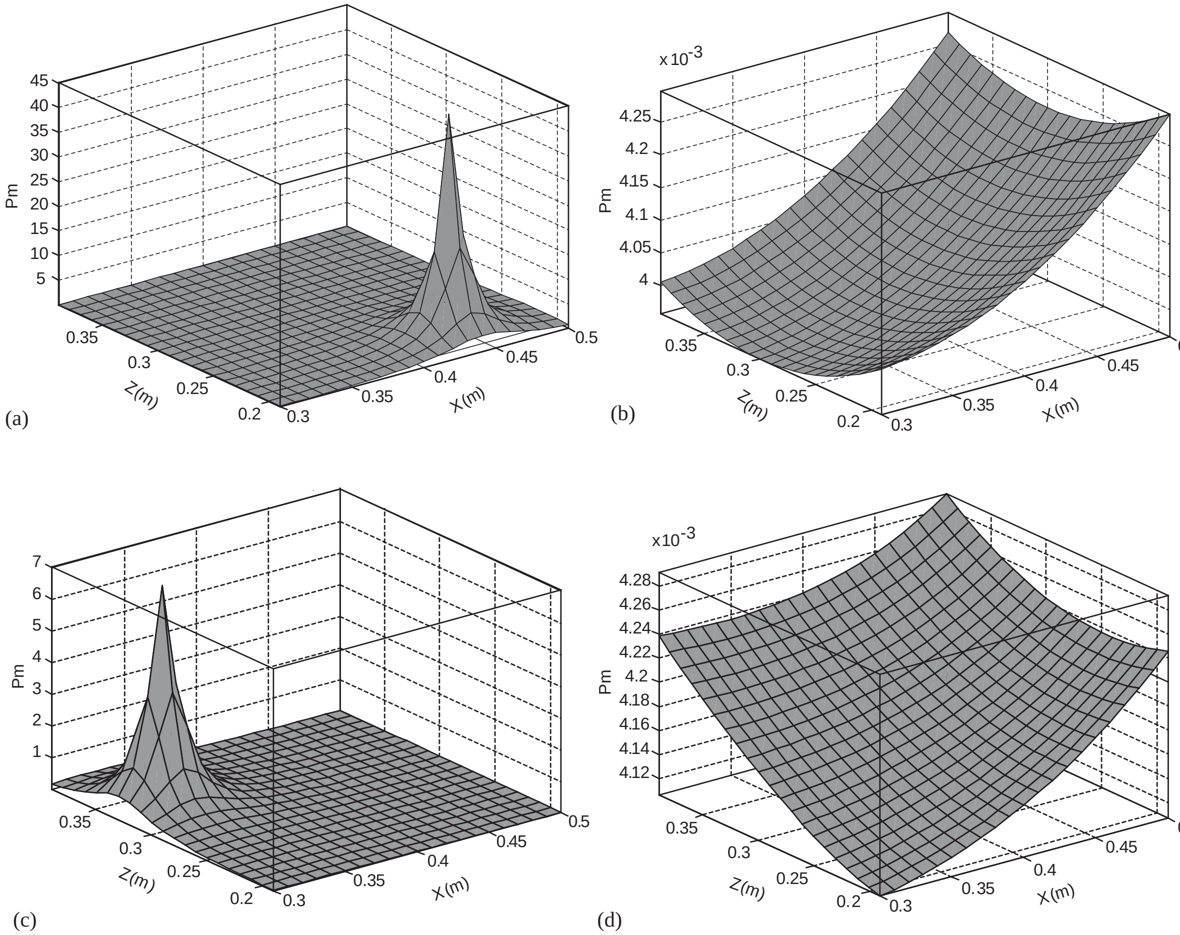

Fig. 10. P_m distributions on the source plane, calculated by the traditional MUSIC method, (a) $f_1 = 410\,\mathrm{Hz}$, (b) $f_2 = 520\,\mathrm{Hz}$, (c) $f_3 = 790\,\mathrm{Hz}$, (d) $f_4 = 1115\,\mathrm{Hz}$.

Table 7
Source positions identified by the traditional MUSIC method, example 2

Frequency (Hz)	Identified source position (*x, y, z*) (m)		
410	S_{11}(0.44, 0.70, 0.22)		
525	No local maximum		
790	S_{31}(0.33, 0.70, 0.33)		
1115	No local maximum		
Summary of result			
Exact (*x, y, z*)	(0.44, 0.70, 0.22)	(0.33, 0.70, 0.33)	(0.22, 0.70, 0.44)
Identified (*x, y , z*)	(0.44, 0.70, 0.22)	(0.33, 0.70, 0.33)	—

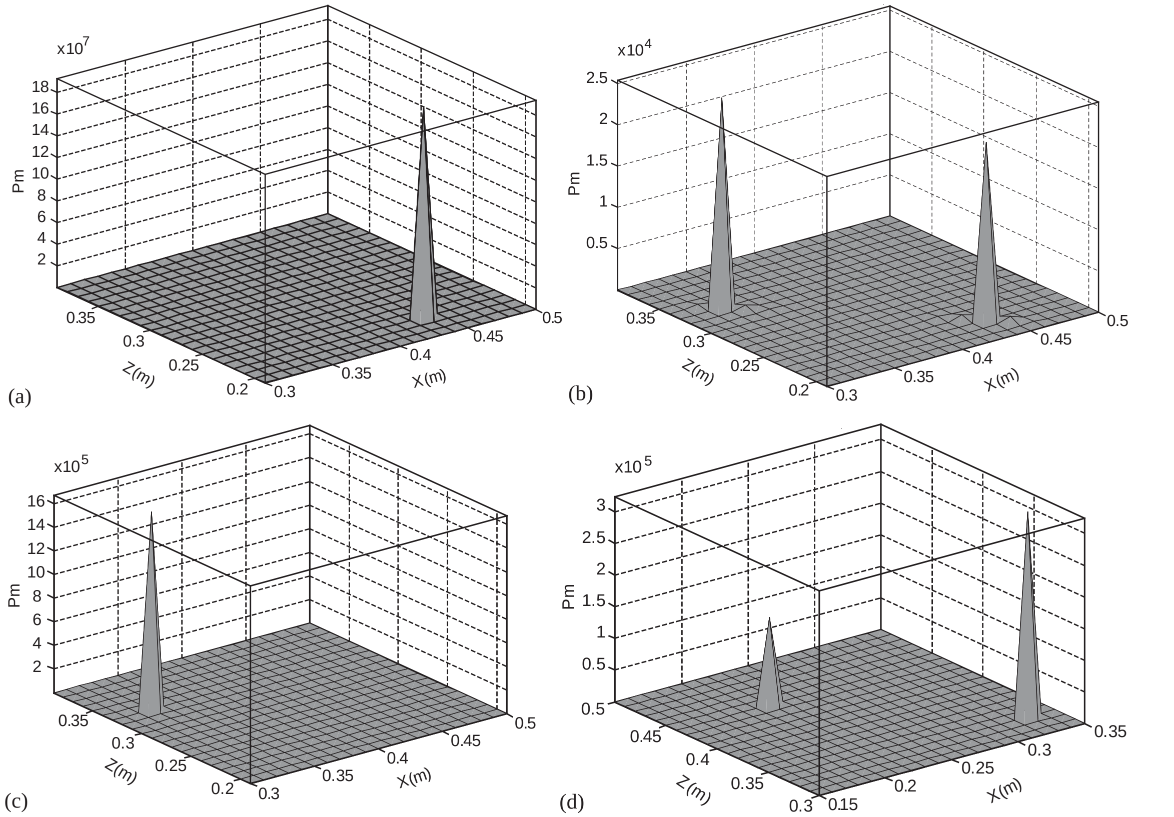

Fig. 11. P_m distributions on the source plane, calculated by the modified method, (a) $f_1 = 410\,\text{Hz}$, (b) $f_2 = 525\,\text{Hz}$, (c) $f_3 = 790\,\text{Hz}$, (d) $f_4 = 1115\,\text{Hz}$.

Table 8
Source positions identified by the modified method, example 2

Frequency (Hz)	Identified source position $(x,\ y,\ z)$ (m)		
410	S_{11}(0.44, 0.70, 0.22)		
525	S_{21}(0.44, 0.70, 0.22), S_{22}(0.33, 0.70, 0.33)		
790	S_{31}(0.33, 0.70, 0.33)		
1115	S_{31}(0.33, 0.70, 0.33), S_{42}(0.22, 0.70, 0.44)		
Summary of result			
Exact $(x,\ y,\ z)$	(0.44, 0.70, 0.22)	(0.33, 0.70, 0.33)	(0.22, 0.70, 0.44)
Identified $(x,\ y,\ z)$	(0.44, 0.70, 0.22)	(0.33, 0.70, 0.33)	(0.22, 0.70, 0.44)

The above two simulated examples demonstrate clearly that

(1) The main problem of the traditional MUSIC method can be solved by the modified method.
(2) The proposed identification procedure is very efficient, only the data at a few frequencies are needed to identify the number and positions of sources.

Here only one- and two- dimensional cases are discussed because it is easier to show the difference between the traditional and the modified methods by figures. The conclusions obtained from the above two examples are valid for three-dimensional case.

The feasibility and accuracy of the proposed method were further verified by experiment, as discussed in the next section.

4. Experimental results and discussions

In this section, the experimental set-up will be described first, and then the experimental results will be discussed.

4.1. Experimental set-up

The experimental set-up is shown in Fig. 12. The measurement instruments include

(1) *Anechoic chamber:* The anechoic chamber was used to create a free-field condition so that the transfer function $\{\bar{h}_j\}$ in Eq. (12) can be obtained accurately by theoretical method.
(2) *Speakers:* Three speakers were used to simulate three-point sources.
(3) *Measurement microphones* (B&K type 4165): Five measurement microphones were used to measure the sound pressure simultaneously.
(4) *Digital data recorders* (Sony PC 208A): Two digital data recorders were used. One recorder was used to record the measured signals; the other recorder was used as player to generate the source signals which were stored in advance in the tape.
(5) *Spectrum analyzer* (HP35670A): Because the number of measurement channels of the spectrum analyzer is only four, the spectrum analyzer was used mainly to monitor the measurement.

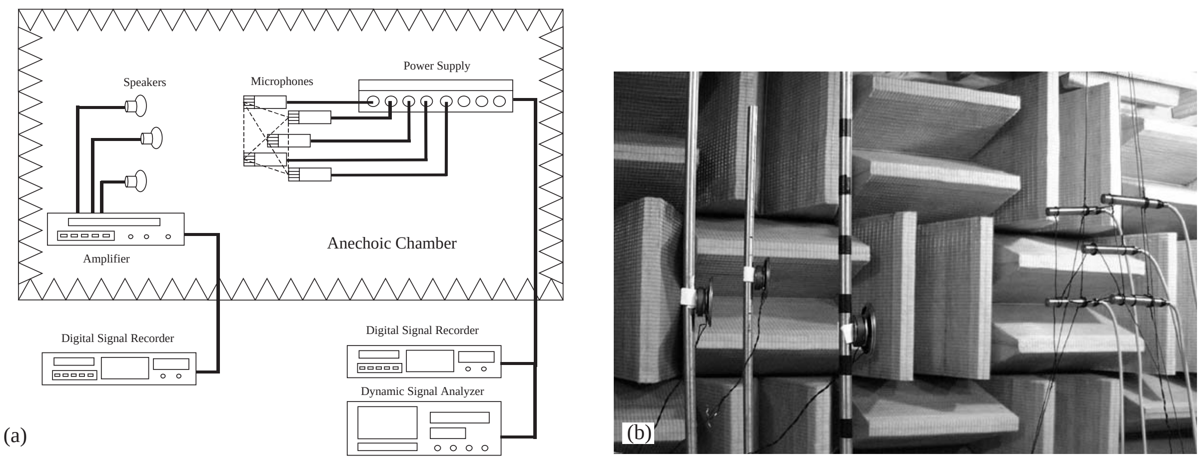

Fig. 12. (a) Experimental set-up, (b) photo of sources and microphones.

Table 9
Positions of sources and microphones of experimental example

	Position vectors	(x, y, z) (m)
Point Sources	$\vec{r}_{s1}$	(0.085, 0.265, 0.235)
	$\vec{r}_{s2}$	(0.185, 0.385, 0.210)
	$\vec{r}_{s3}$	(0.300, 0.335, 0.300)
Measurement microphones	$\vec{r}_{m1}$	(0.080, 0.020, 0.280)
	$\vec{r}_{m2}$	(0.280, 0.020, 0.280)
	$\vec{r}_{m3}$	(0.180, 0.020, 0.180)
	$\vec{r}_{m4}$	(0.080, 0.020, 0.080)
	$\vec{r}_{m5}$	(0.280, 0.020, 0.080)

The three source signals are narrow band random signals with central frequencies at 800, 2000 and 4000 Hz, respectively.

4.2. *Results and discussions*

The positions of sources and microphones are listed in Table 9. The envelope of the average pressure spectrum of the five microphones is shown in Fig. 13, and the frequencies of the local maxima are $f_1 = 800$ Hz, $f_2 = 2000$ Hz and $f_3 = 3900$ Hz. The truncated rank distribution is shown in Fig. 14, and the frequencies of the local maxima are $f_4 = 800$ Hz, $f_5 = 2800$ Hz, $f_6 = 3900$ Hz. Because f_1 and f_3 are equal to f_4 and f_6, only the data at four frequencies should be used to identify the number and locations of sources. One cannot know the measurement noise in this case. However, the eigenvalues of Eq. (7b) can give us some information about the measurement noise. The normalized eigenvalues at frequencies 800, 2000, 2800 and 3900 Hz are listed in Table 10. As indicated in Eq. (9), if the measurement noise levels are identical, then the noise-eigenvalues should be repeated. For instance, there are only two eigenvalues with significant value (i.e., larger than 10^{-6}) at frequency 2800 Hz. The other three insignificant eigenvalues are not equal. That means the measurement noise levels are not equal in each measurement channel at frequency 2800 Hz. The result of Table 10 indicates that the measurement noise levels generally are not equal in each measurement channel in practice. To identify the source positions, a space which includes the three sources were divided in to $80 \times 80 \times 80 = 512{,}000$ trial field points with a spatial resolution of 0.5 cm. The result identified by the traditional MUSIC method is shown in Table 11. One can find that the result is completely false, i.e., no local maximum was found in the searched space. This result is expected because the result of Table 10 indicates that the basic assumption of the traditional MUSIC method cannot be fulfilled in this case.

To expose the problem of the traditional MUSIC method more clearly, the P_m distribution is shown here. Because the local maxima of the P_m distribution is very difficult to show in three-dimensional case, only a typical two-dimensional distribution is shown here. Fig. 15 shows the P_m distribution on the plane $Z = 0.300$ on which the source 3 is located. The result was calculated

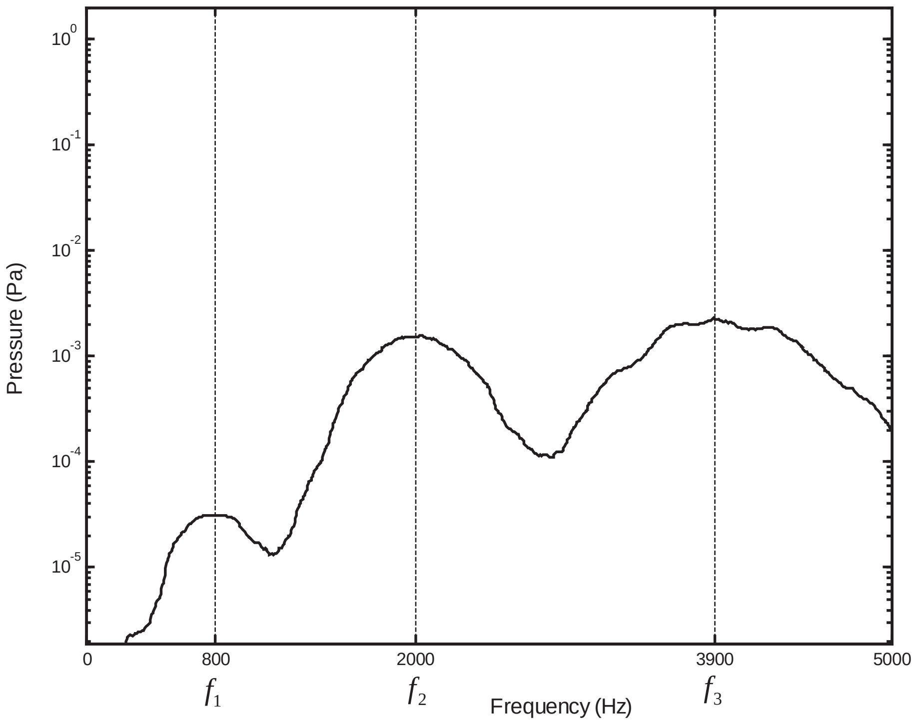

Fig. 13. Envelope of the average spectrum of experimental example.

with the data at 800 Hz which was the central frequency of source 3. Theoretically, a local maximum of the P_m distribution should be found at or near $x = 0.300$, $y = 0.335$ because the source 3 is located at that point. However, the P_m distribution found by the traditional MUSIC method does not show that characteristic. On the contrary, the P_m distribution found by the modified MUSIC method is shown in Fig. 16. One can find clearly a local maximum at $x = 0.300$ and $y = 0.335$. The above results of P_m distributions show clearly the difference between the traditional and the modified MUSIC methods. The source positions identified by the modified method are listed in Table 12. As mentioned, the mesh resolution of the trial field points used to calculate the P_m values is 0.5 cm, the identified positions with a difference equal to the mesh resolution are considered as the same point. For instance, one of the source positions found at 2800 Hz is (0.185, 0.390, 0.210) and one of the source positions found at 3900 Hz is (0.185, 0.385, 0.210). Because the difference between these two positions is equal to the mesh resolution used for calculation, these two positions are considered as the same position. The result of Table 12 shows that the source positions can be accurately identified by the modified MUSIC method. The result of Table 12 also indicates that the number of sources is three.

The above experimental results demonstrate that the proposed modified MUSIC method is feasible in practice and show that the identification accuracy of the modified method is far better than the traditional method. The maximum error identified by the

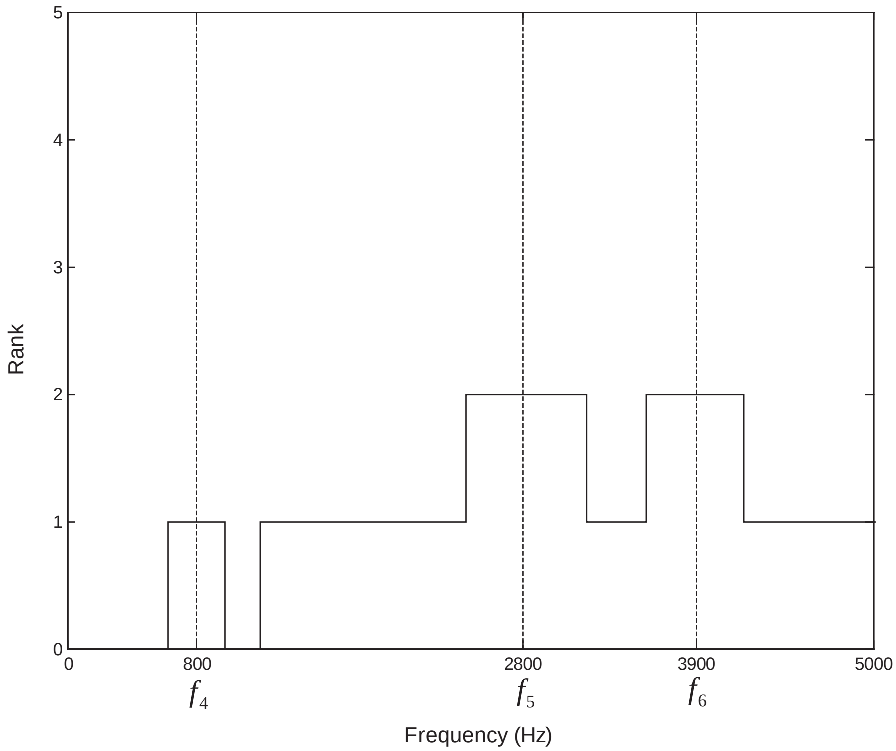

Fig. 14. The true rank distribution of experimental example.

Table 10
Normalized eigenvalues at the selected frequencies

	800 Hz	2000 Hz	2800 Hz	3900 Hz
λ_1	2.0e-4	0.07	1.9e-3	1
λ_2	1.8e-8	5.2e-7	6.6e-5	2.0e-5
λ_3	1.6e-10	1.8e-8	7.3e-10	7.1e-7
λ_4	9.5e-11	2.9e-10	2.4e-10	9.8e-9
λ_5	6.8e-11	1.5e-10	1.1e-10	1.9e-9

modified method is equal to the mesh resolution. The experimental results also demonstrate that the proposed calculation procedure is very efficient, only the data at a few frequencies should be used for identification.

5. Conclusions

The identification of the number and location of acoustic sources is a fundamental problem of engineering acoustics. The MUSIC method is an important method to solve this

Table 11
Source positions identified by the traditional MUSIC method

Frequency (Hz)	Identified source position (*x, y, z*) (m)		
800	No local maximum		
2000	No local maximum		
2800	No local maximum		
3900	No local maximum		
Summary of result			
Exact (*x, y, z*)	(0.22, 0.70, 0.22)	(0.33, 0.70, 0.33)	(0.22, 0.70, 0.44)
Identified (*x, y, z*)	—	—	—

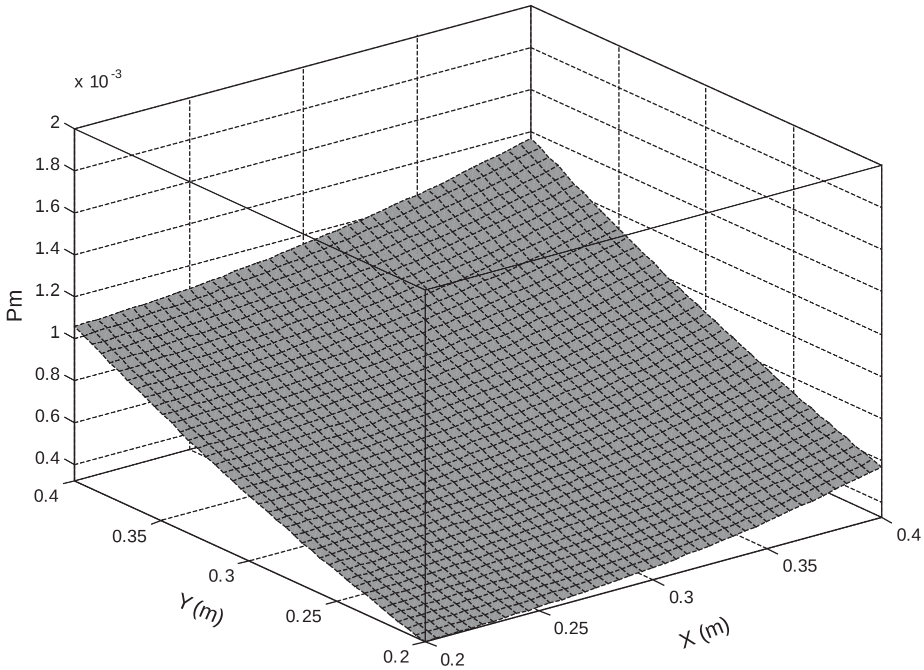

Fig. 15. P_m distribution from the traditional MUSIC method, on the plane $z = 0.300$, 800 Hz.

problem. However, the tradition MUSIC method functions well only when the unavoidable measurement noise is identical for all the measurement channels. In this work, the traditional MUSIC method was modified to eliminate the basic requirement of the method. Because the modified method should adjust the measured noise levels in frequency domain, a calculation procedure was proposed to select only a few ''important'' frequencies for identification in order to enhance the calculation efficiency. The numerical and experimental results demonstrated that the identification accuracy of the proposed method was far better than that of the traditional MUSIC method. The results also proved that the

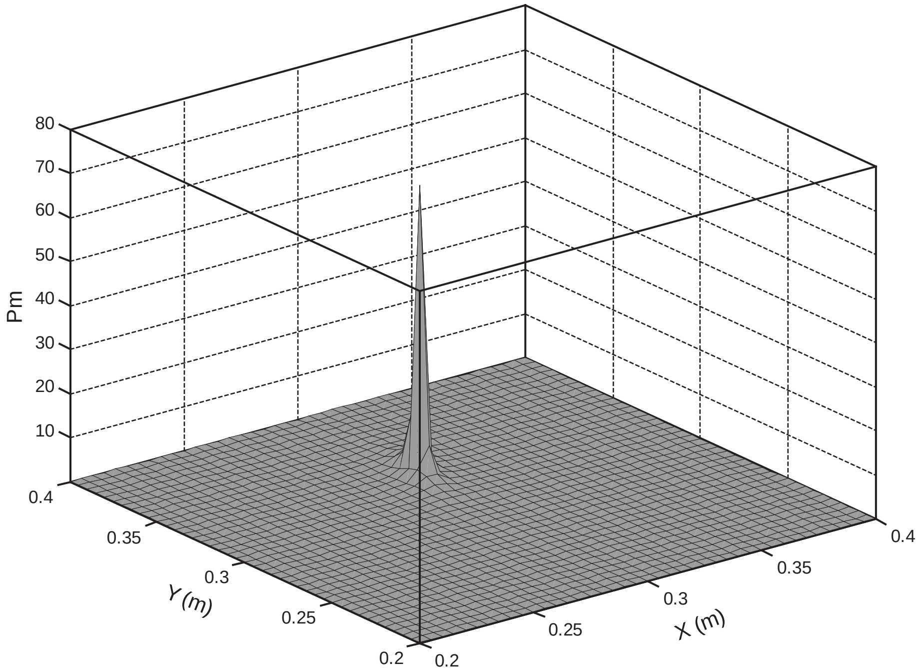

Fig. 16. P_m distribution from the modified method, on the plane $z = 0.300$, 800 Hz.

Table 12
Source positions identified by the modified method

Frequency (Hz)	Identified source position (*x, y, z*) (m)		
800	S_{11}(0.30, 0.300, 0.300)		
2000	S_{21}(0.185, 0.390, 0.210)		
2800	S_{31}(0.185, 0.390, 0.210), S_{32}(0.085, 0.265, 0.235)		
3900	S_{41}(0.185, 0.385, 0.210), S_{42}(0.085, 0.265, 0.235)		
Summary of result			
Exact (*x, y, z*)	(0.085, 0.265, 0.235)	(0.185, 0.385, 0.210)	(0.300, 0.335, 0.300)
Identified (*x, y, z*)	(0.085, 0.265, 0.235)	(0.180, 0.385–0.390, 0.210)	(0.300, 0.335, 0.300)

proposed calculation procedure was feasible and efficient in practice, only the data at a few frequencies should be used for identification.

Acknowledgements

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Appendix A

The "Property A" stated in Section 2.3 was proven in this Appendix. One should prove that result (b) in Eq. (15) is correct. It is very difficult to prove the result (b) in Eq. (15) directly by a closed-form formulation. The validity of the result (b) in Eq. (15) was proven indirectly by using some properties of eigenanalysis of the $[C_p]$ matrix. For simplicity, the number of active sources (or the true rank) was assumed as n in the following derivation. The properties are formulated and proven in what follows.

Property 1. Only when the measurement noise levels at all the measurement channels are identical, the noise-eigenvalues are repeated (or identical).

Proof. This is the basic result of the traditional MUSIC method [10], as indicated in Eq. (9). □

Property 2. If the measurement noise levels are not identical, then the noise -eigenvalues will not be repeated.

Proof. This is an equivalent statement of Property 1. □

Property 3. If an arbitrary positive value δ is added to the kth diagonal term of the $[C_p]$ matrix which contains the largest measurement noise, then $\bar{\lambda}_{n+1}/\bar{\lambda}_m < \lambda_{n+1}/\lambda_m$ is impossible.

Proof. It is assumed that the kth diagonal term of the $[C_p]$ matrix contains the largest measurement noise, and the positive value δ is added to that term. If $\bar{\lambda}_{n+1}/\bar{\lambda}_m < \lambda_{n+1}/\lambda_m$ is possible, then that means one can add noise repeatedly to the kth diagonal term of the $[C_p]$ matrix to make the ratio $\bar{\lambda}_{n+1}/\bar{\lambda}_m$smaller and smaller (i.e., chose to one). In the limited case, one can add noise to the kth diagonal term to make the ratio $\bar{\lambda}_{n+1}/\bar{\lambda}_m = 1$. This result violates the Property 2.

Thus, $\bar{\lambda}_{n+1}/\bar{\lambda}_m < \lambda_{n+1}/\lambda_m$ is impossible if a positive value is added to the kth diagonal term of the $[C_p]$ matrix which contains the largest noise level. □

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